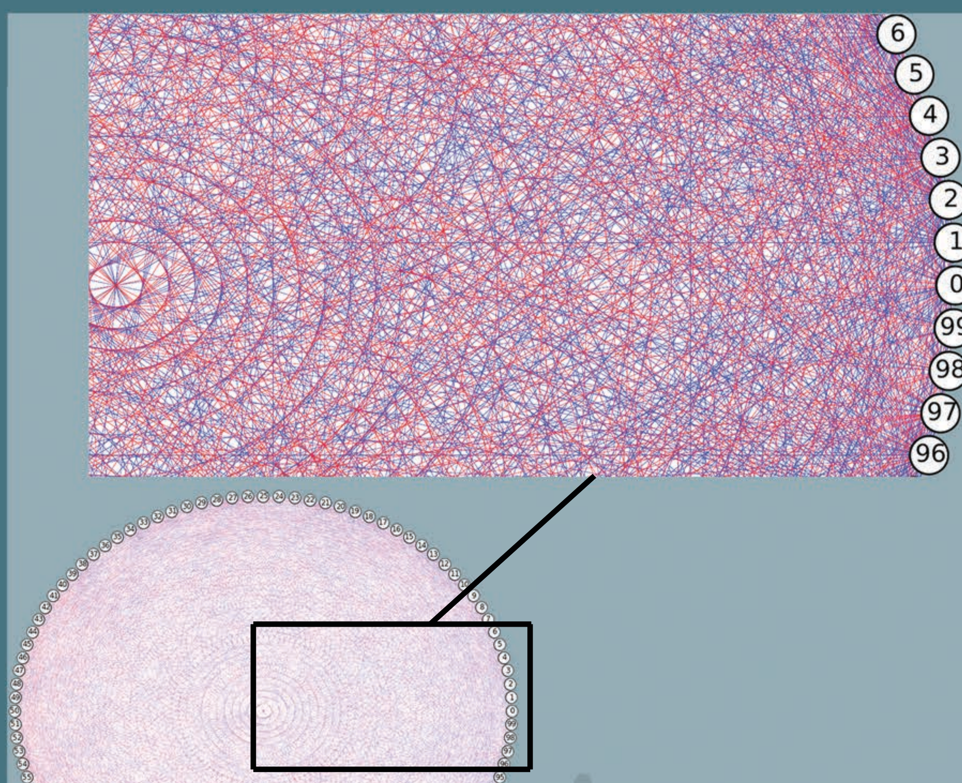
Is the Enemy of an Enemy, a Friend?

structural Balance in Signed Networks

Background
Theories of social structure are connected to network science tools and techniques to study local behaviours and global structures in social communities with relations of opposite nature. Such communities are modelled by graphs with positive and negative edges called signed graphs. The building block of structural balance theory is a work by Heider in 1944 that was expanded into a set of graph-theoretic concepts by Cartwright and Harary (1956) to handle a social psychology problem. According to their definitions, a tie to a distant neighbour can be expressed by the product of signs on edges reaching them. The product of signs in cycles exhibits their balance status. Total balance is guaranteed for signed networks containing no cycles with an odd number of negative edges.

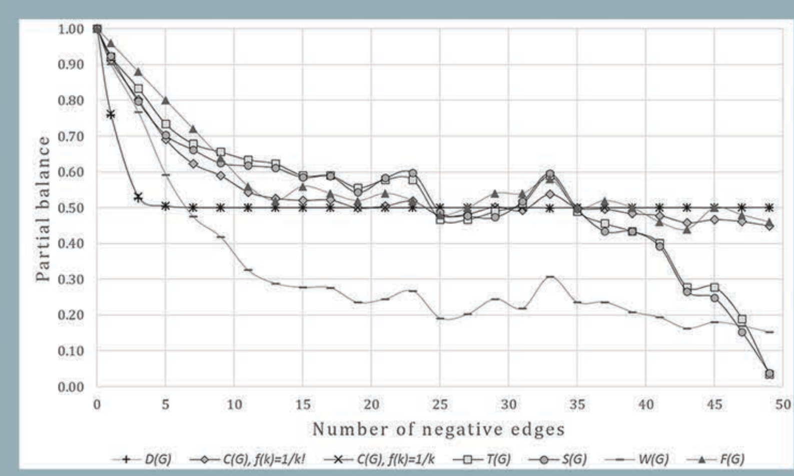
Problem Statement
However, it is quite unlikely for a signed network to be in a total balance. We are interested in measuring how far a network is from balance. Structural balance is widely used in many applications such as predicting positive and negative edges. Some predictions provided by this theory are controversial. The dynamics leading to specific global structures in signed networks remain speculative unless a framework is provided to test such dynamics.

Our contribution
We provide a numerical comparison of several measures of partial balance on a variety of undirected signed networks and random graphs. Using some theoretical results for simple classes of graphs, we suggest a framework to evaluate these measures and shed light on the context-dependency involved in using them.

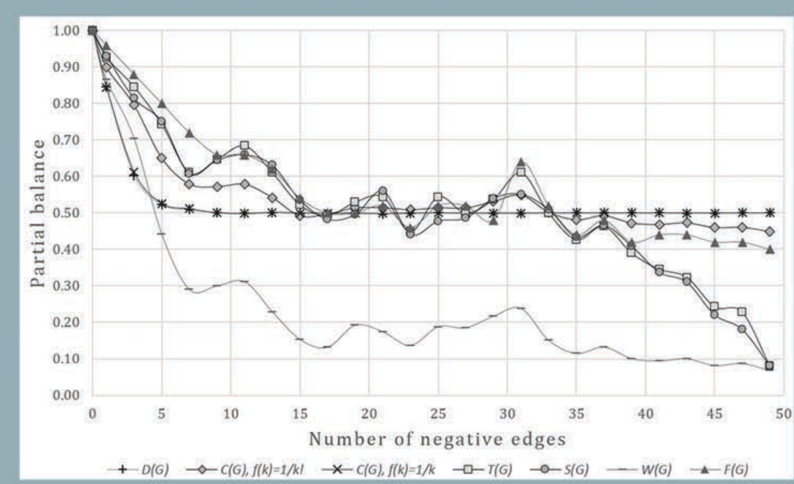


Dataset/Graph	n	m	m−	T(G)	S(G)	F(G)
Senators	100	2461	1047	0.86	0.84	0.73
G(n,m)				0.51	0.52	0.22

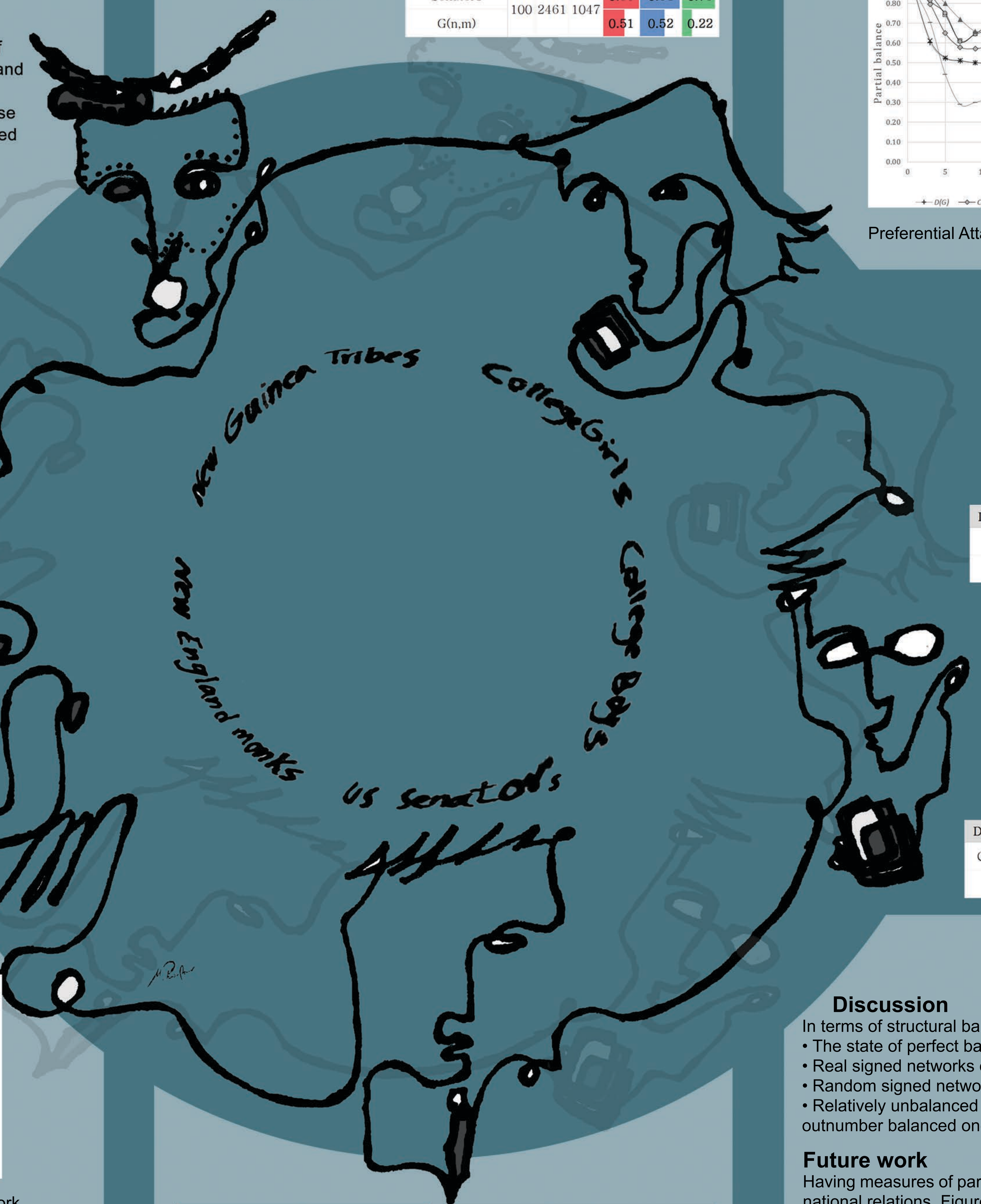
Synthetic datasets
Balance is analysed in randomly generated graphs with different number of negative edges. Figures demonstrate the partial balance in random networks measured by different methods.

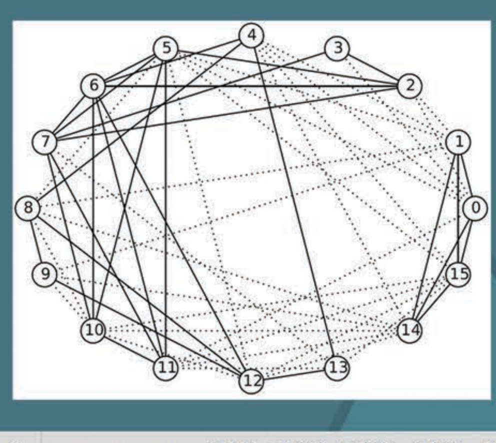


Erdos-Renyi (n=15, p=0.45) network with 50 edges

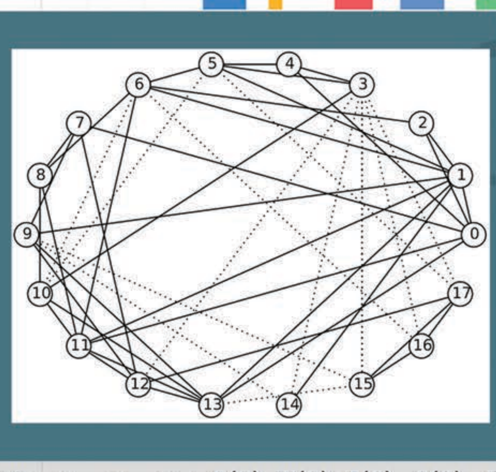


Preferential Attachment (n=15,m=5) network with 50 edges

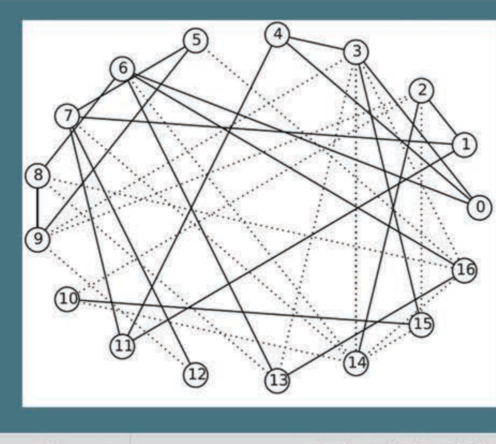




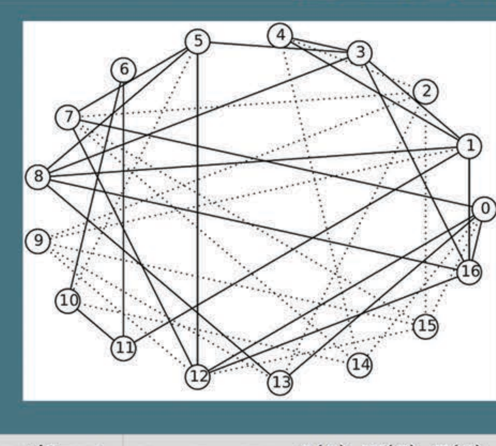
Dataset/Graph	n	m	m−	C(G)	W(G)	T(G)	S(G)	F(G)
Tribes	16	58	29	0.68	0.53	0.87	0.88	0.76
G(n,m)				0.50	0.17	0.52	0.52	0.45



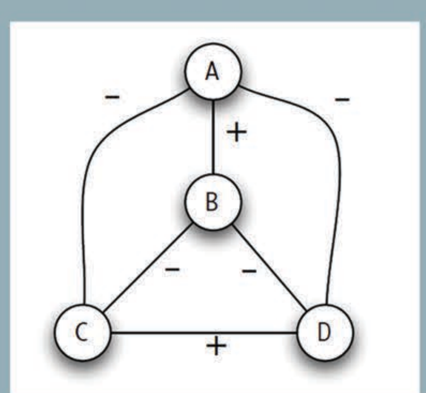
Dataset/Graph	n	m	m−	C(G)	W(G)	T(G)	S(G)	F(G)
Monks	18	49	12	0.72	0.74	0.86	0.84	0.80
G(n,m)				0.54	0.50	0.57	0.55	0.63



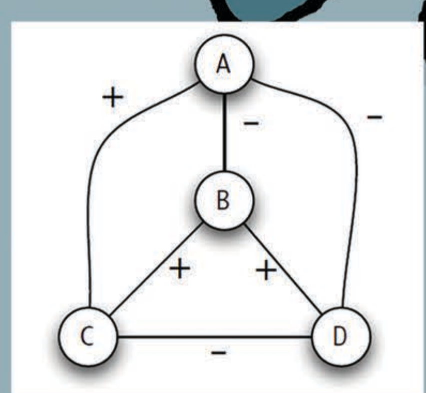
Dataset/Graph	n	m	m−	C(G)	W(G)	T(G)	S(G)	F(G)
College Girls	17	36	16	0.62	0.76	0.79	0.84	0.67
G(n,m)				0.44	0.60	0.42	0.57	0.61



Dataset/Graph	n	m	m−	C(G)	W(G)	T(G)	S(G)	F(G)
College Boys	17	40	17	0.74	0.83	0.78	0.80	0.80
G(n,m)				0.51	0.69	0.50	0.49	0.65



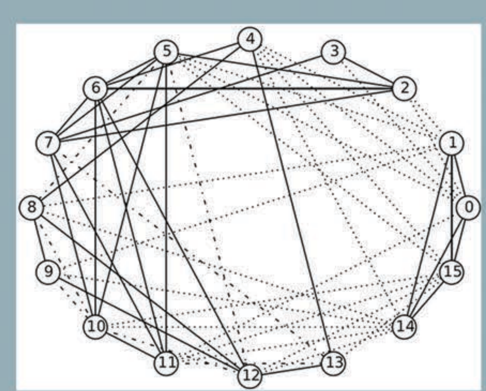
A completely balanced signed network where all cycles are positive (Easley and Kleinberg 2010 pp 122)



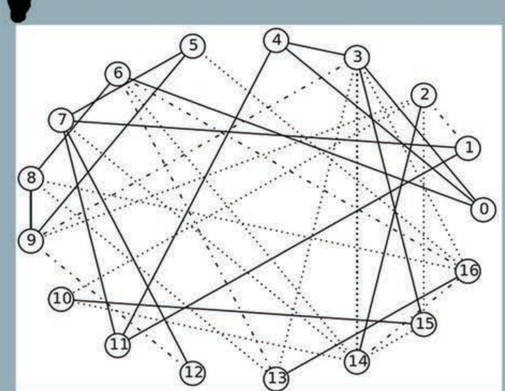
An unbalanced signed network with positive and negative cycles (Easley and Kleinberg 2010 pp 122)

Method
Measures of partial balance are deployed to differentiate networks quantitatively with respect to their level of balance.
Degree of balance D(G) (Cartwright and Harary 1956):
A computationally exhaustive cycle-based measure
Weighted degree of balance C(G) (Norman 1972):
Using cycles weighted by a decreasing function of the length
Walk-based degree of balance W(G) (Estrada 2014):
A simplified measure replacing cycles by closed-walks
Triangle index T(G) (Terzi 2011):
A more simplified measure considering only closed walks of length 3
Relative signed clustering coefficient S(G) (Kunegis 2014):
Sampling over triads and normalising by clustering coefficient
Normalised frustration index F(G):
Normalised minimum number of edges whose removal results in balance (Harary 1959)

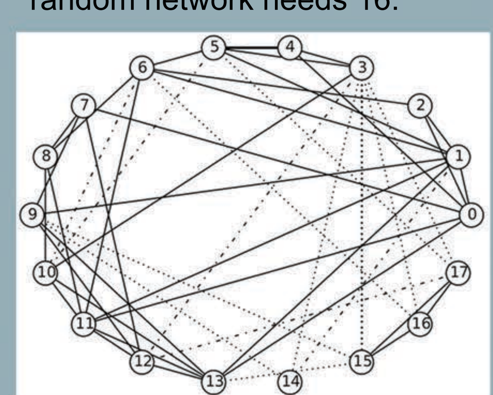
Real-world datasets
Graphs in the middle of poster demonstrates four signed networks where dotted lines represent negative edges and solid lines represent positive ones.
The measures of partial balance are computed for signed networks and their similar random graphs. Numerical results show that real signed networks are more inclined towards balance than expected by random.
It is interesting to see how these small signed networks can become balanced by removing a few edges. These edges are represented by dot-dash lines in the graphs. The number of such edges are less than what we have in random networks.



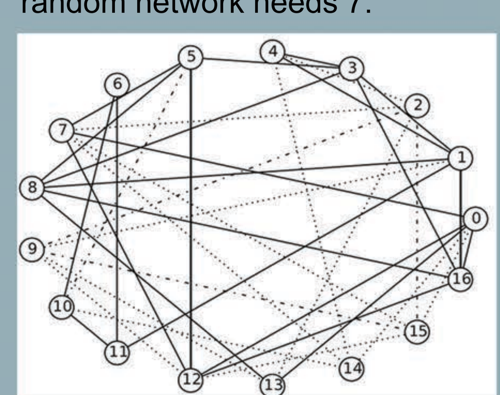
Highland tribes network becomes balanced after removing 7 negative edges. A similar random network needs 16.



College girls network becomes balanced after removing 3 positive and 3 negative edges. A similar random network needs 7.



Monastery interactions network becomes balanced after removing 2 positive and 3 negative edges. A similar random network needs 9.

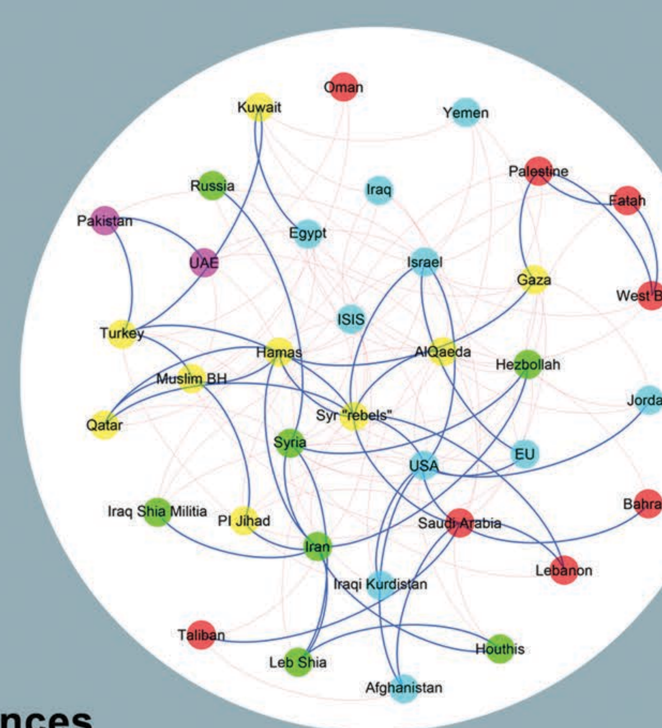


Fraternity preferences network becomes balanced after removing 4 negative edges. A similar random network needs 7.

Findings
Real signed networks are more balanced than we expect by random.
Networks analysed have mostly transitive relationships.
The enemy of your enemy is more of a friend than an enemy. (No matter you are a member of a New Guinea tribe, a novice monk in New England monastery, a girl in an Eastern collage, a boy in an American fraternity, or a US senator)

Discussion
In terms of structural balance we can suggest an order as follows.
- The state of perfect balance where all cycles are balanced
- Real signed networks of human interactions and preferences
- Random signed networks with non-trivial number of negative edges
- Relatively unbalanced structures where unbalanced cycles outnumber balanced ones

Future work
Having measures of partial balance in hand, we plan to analyse international relations. Figure shows clustering analysis done on the signed network of Middle East key players.



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NetSci Wrocław
International School and Conference on Network Science | Poland, January 11–13, 2016

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